# PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

SEPTEMBER, 1955



DISCUSSION OF PROCEEDINGS PAPERS

529, 670, 671, 677

ENGINEERING MECHANICS
DIVISION

Copyright 1955 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society 33 W. 39th St. New York 18, N. Y.

PRICE \$0.50 PER COPY

Current discussion of papers sponsored by the Engineering Mechanics Division is presented as follows:

Number		or .	Page
	529	Beam Restraints Provided by Walls with Openings, by I. A. Mohammed and E. P. Popov. (October, 1954. Prior discussion: None. Discussion closed)	
		Corrections	1
	670	Virtual Mass and Acceleration in Fluids, by T. E. Stelson and F. T. Mavis. (April, 1955. Prior discussion: None. Discussion closed)	
		Caldwell, Joseph M	3
	671	A Resistor-Network Solution for the Elasto-Plastic Torsion Problem, by J. H. Weiner, M. G. Salvadori, and V. Paschkis. (April, 1955. Prior discussion: None. Discussion closed)	
		Moore, Howard R	7
	677	Vessels Partially Supported by Soil, by W. A. Boothe, R. T. Gray, and G. Horvay. (April, 1955. Prior discussion: None. Discussion closed)	
		Corrections	9

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference (or paper number), and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

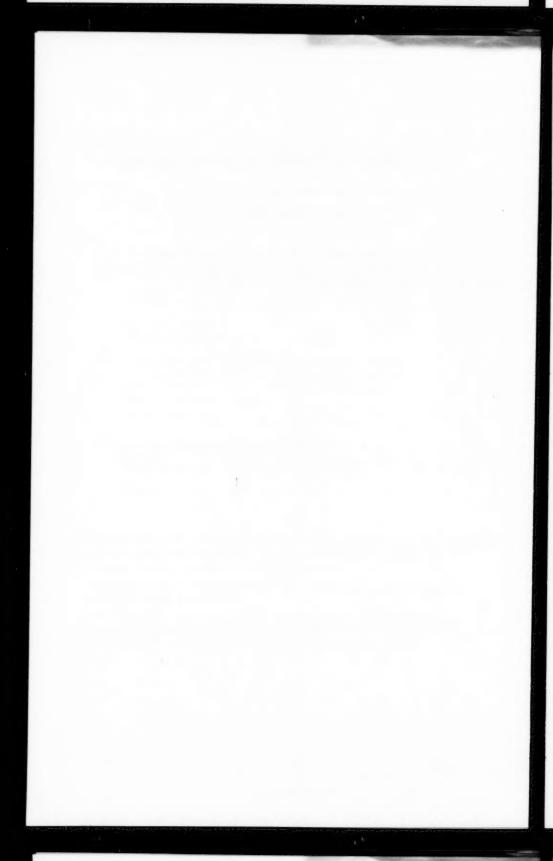
This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N.Y.

## Discussion of "BEAM RESTRAINTS PROVIDED BY WALLS WITH OPENINGS"

by I. A. Mohammed and E. P. Popov

(Proc. Paper 529)

CORRECTIONS.—Line 4, page 13, should read: "The deflection w for  $\eta \leq y \leq \eta_2$  is:." The last line of mathematics on page 12 should be transposed to follow line 4 on page 13, forming the beginning of Eq. 8.



### Discussion of "VIRTUAL MASS AND ACCELERATION IN FLUIDS"

by T. E. Stelson and F. T. Mavis

(Proc. Paper 670)

JOSEPH M. CALDWELL, M. ASCE.—The authors are to be commended for presenting in a readily understandable manner the basic concept of virtual mass and demonstrating its potential importance in engineering practice. Possibly the value of the experimental data diagrammed in the paper could be

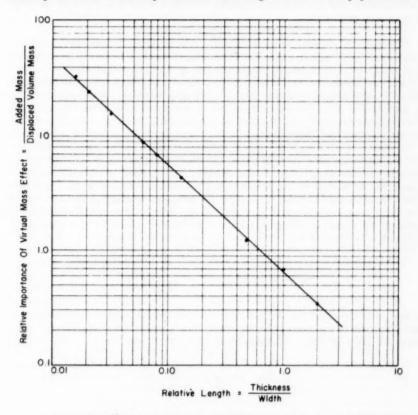


FIGURE FRELATIVE EFFECT OF VIRTUAL MASS IN PARALLELEPIPEDS WITH SQUARE SIDE MOVING BROADSIDE - ON

Chief, Research Div., Beach Erosion Board, Corps of Engrs., Washington, D. C.

further increased by a replot of the data shown in Figure 4 of the paper. A first look at Figure 4, unless accompanied by a careful reading of the text, might give the impression that the relative importance of the virtual mass effect increases as the thickness, or "length," of the object increases, while in fact the relative importance decreases.

The decrease in relative importance of the virtual mass can be demonstrated by replotting the data with the ratio of "added mass" to "displaced volume mass" as the parameter for the axis of ordinates. The accompanying plot (Figure 5) shows the result of such a presentation of the data; the decreasing relative importance of the virtual mass effect as the object becomes

more and more elongated in the direction of motion is apparent.

A possible worthwhile extension of the author's work would be to carry it into the problem of the fixed object held in an accelerating fluid flow. This latter problem is being faced with increasing frequency in the design of offshore structures subjected to wave action around the continental shelf of the United States, and the concepts of virtual mass are not yet clarified in many such design problems.

EDWARD SILBERMAN, A.M. ASCE. - The method for measuring virtual mass introduced in this paper should prove extremely useful. The carefulness with which the work was done and the agreement with theory where applicable indicates that the results presented are of a high order of accuracy. However, the statement in the introduction that "determination of added mass by experiment often has been inconsistent and unreasonable" and the associated implication that results not in agreement with those of the authors are in error require further proof.

It should be observed that in the references cited by the authors, as involving added masses several times the theoretical values, (3),(4) the bodies were in unidirectional, rather than in vibratory motion. A body in unidirectional motion sooner or later develops a boundary layer and wake. The question then arises as to what part of the resistance to motion is due to viscous and form drag and what part to added mass. The question can be best studied, perhaps, by reducing all resistance or drag values to the form of a drag coefficient, Cn defined by

Resistance = 
$$C_D \rho S \frac{v^2}{2}$$

where  $\rho$  is the fluid density and S is the area of maximum cross section taken normal to the direction of motion. For a sphere of diameter D, for example, the resistance attributable to added mass is

with 
$$c \frac{dv}{dt} \approx \frac{1}{2} \rho \pi \frac{D^3}{6} \frac{dv}{dt}$$

$$S = \frac{\pi D^2}{4}$$

$$C_D = \frac{2}{3} \frac{D}{v^2} \frac{dv}{dt}$$

a. Associate Prof., St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn.

In Figure 6 are plotted some experimental data for spheres in unidirectional acceleration.  $^{\rm b}$  The preceding equation for the drag coefficient attributable to added mass is also shown in the figure, as is the drag coefficient for unaccelerated motion. The abscissa is the dimensionless parameter  $(D/v^2)$  dv/dt used by Iverson and Balent.  $^{\rm (4)}$  This figure is typical of unidirectional acceleration data; the data of Iverson and Balent for circular disks plot in similar manner, for example.

It is apparent that the precision in unidirectional acceleration measurements is not nearly so great as that in the authors' experiments. Nevertheless, the results are sufficiently consistent when plotted in this fashion to indicate a definite trend. A first attempt toward predicting the fluid resistance in unidirectional acceleration would be to add the resistance attributable to the added mass of the body to the resistance attributable to viscous motion at the instantaneous Reynolds number of the motion. This has been tried and the result is shown by the broken line in the accompanying figure. It is apparent that this simple approach fails; a similar result occurs for all other unidirectional acceleration data known to the writer.

There are two or three other possibilities for predicting the resistance in unidirectional acceleration. These are: (1) The viscous drag corresponding to a Reynolds number much smaller than its instantaneous value should be used. (2) The added mass for a body longer than the actual body, to allow for the presence of the wake, should be used. (3) A combination of smaller Reynolds number and larger added mass should be used. (In support of the second choice, above, the authors' Fig. 4 shows that as the length of a parallelepiped increases, its added mass with respect to the displaced mass of a cube increases. If the wake behind a cube could be considered as adding to its length, the cube plus wake combination would have a larger added mass and larger resistance than the cube alone.) The writer is inclined to believe that either choice (2) or (3), above, is the proper explanation for the trend of the data and that the added mass is actually a function of the state of the motion. Only at very large values of the parameter  $(D/v^2)$  dv/dt (i.e. in the absence of a wake) do the authors' results apply to unidirectional motion.

Regarding the importance of added mass in low-density fluids, it should be noted that the accompanying figure is largely independent of the fluid used. Hence, even in air, at high values of the parameter  $(D/v^2) \, dv/dt$ , the resistance attributable to added mass may be much more important than the viscous and form drag associated with the instantaneous Reynolds number of the motion.

It is interesting to observe in connection with the authors' Fig. 3 that the data for the rectangular plates can be used to extend the plotting to the right of the length-width ratio of unity. That is, the added mass for a plate of length-width ratio of 4, say, must be the same as for a plate of the same area of length-width ratio 1/4, assuming no scale effect. The displaced cylinder masses in the example cited are in the ratio of 1:16. Thus, the ordinate occurring at the length-width ratio of 4 may be divided by 16 and plotted at the length-width ratio of 1/4. Proceeding similarly with all the data, the straight line drawn through the rectangular plate data on Fig. 3 is seen to be a curve asymptotic to the ordinate zero. (Actually, the curve and straight line are nearly identical in the region of Fig. 3.) By inference, the circular cylinder data may be assumed to follow a similar trend.

b. Bugliarello, G. "The Resistance to Accelerated Motion of Spheres in Water." Thesis submitted to the Graduate Faculty of the University of Minnesota, July 1954. Library, University of Minnesota. 131 pages.

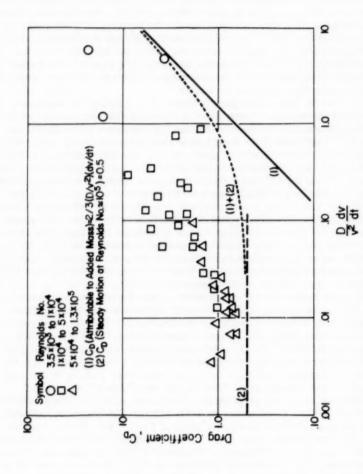


FIG. 6 Drag Coefficient for 3in Steel Spheres Accelerated Unidirectionally

## Discussion of "A RESISTOR-NETWORK SOLUTION FOR THE ELASTO-PLASTIC TORSION PROBLEM"

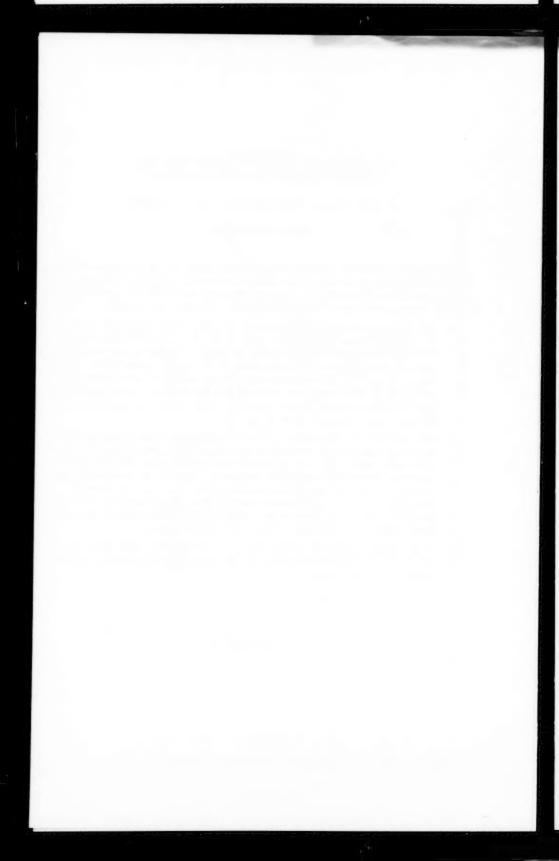
by J. H. Weiner, M. G. Salvadori, and V. Paschkis

(Proc. Paper 671)

HOWARD R. MOORE. 1—The authors of this paper are to be congratulated on their ingenious adaptation of the resistor-network technique to handle the elasto-plastic torsion problem. Several specific comments and questions occurred to the writer when reading this paper. They are as follows:

- 1. The authors point out the importance of obtaining an accurate value of the dimensionless slope of the roof,  $\tau_o/G$ . It occurred to the writer that the following procedure might be helpful. Perform two "elastic" network experiments with mesh sizes of 2h and h respectively. Using values of  $e_i$  from these experiments it is possible to extrapolate to values of  $e_i$  which would be obtained if a much finer net had been used. These values obtained by extrapolation could be used in equation 9 to obtain a more accurate value of  $\tau_0/G$ .
- 2. The extension of the resistor-network technique to solve this problem raises the question as to its extension into other areas where it has not previously been used, i.e. into problems described by Poisson-type equations plus other analytical conditions. Might it be possible to use "tricks" similar to the one employed here to solve such problems? Distribution of thermal stresses and the flow of not perfectly viscous fluids in conduits of constant but arbitrary cross section are two problems which come to mind. There may well be others.
- 3. A successful method of measuring  $I_0$  I would reduce the labor considerably, as the authors point out. Is further work being done on this aspect of the problem?

<sup>1.</sup> Superv. Engr., Research Dept., Caterpillar Tractor Co.



#### Discussion of "VESSELS PARTIALLY SUPPORTED BY SOIL"

by W. A. Boothe, R. T. Gray, and G. Horvay

(Proc. Paper 677)

CORRECTIONS.-The second line above Eq. 1a should read

The ninth line below Eq. 3 should read

. . . . as derived from Fig. 6 for  $\sigma_x$  and a . . . .

The eleventh line below Eq. 3 should contain a closing parenthesis immediately following the term  $\sigma_{\frac{\partial}{\partial 21}i}$ .

Eq. 8 should be:  $\rho = 225/520 = 0.43$ .

Eq. 14 should contain a diagonal between & and R.

In the line immediately above Eq. 19, the colon should be a period, and in the line above Eq. 21, "not" should be "no."

In footnote 6, "in press" should be changed to: "Vol. 22, p. 25, 1955."

#### AMERICAN SOCIETY OF CIVIL ENGINEERS

#### OFFICERS FOR 1955

#### PRESIDENT WILLIAM ROY GLIDDEN

#### VICE-PRESIDENTS

Term expires October, 1955: ENOCH R. NEEDLES MASON G. LOCKWOOD

Term expires October, 1956: FRANK L. WEAVER LOUIS R. HOWSON

#### DIRECTORS

Term expires October, 1955: CHARLES B. MOLINEAUX WILLIAM S. LaLONDE, JR. JEWELL M. GARRELTS MERCEL J. SHELTON A. A. K. BOOTH CARL G. PAULSEN LLOYD D. KNAPP GLENN W. HOLCOMB FRANCIS M. DAWSON

Term expires October, 1956: Term expires October, 1957: OLIVER W. HARTWELL THOMAS C. SHEDD SAMUEL B. MORRIS RAYMOND F. DAWSON

FREDERICK H. PAULSON GEORGE S. RICHARDSON DON M. CORBETT ERNEST W. CARLTON GRAHAM P. WILLOUGHBY LAWRENCE A. ELSENER

PAST-PRESIDENTS Members of the Board

WALTER L. HUBER

DANIEL V. TERRELL

EXECUTIVE SECRETARY WILLIAM H. WISELY

TREASURER CHARLES E. TROUT

ASSISTANT SECRETARY E. L. CHANDLER

ASSISTANT TREASURER CARLTON S. PROCTOR

#### PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN Manager of Technical Publications

DEFOREST A. MATTESON, JR. Editor of Technical Publications

PAUL A. PARISI Assoc. Editor of Technical Publications

#### COMMITTEE ON PUBLICATIONS

SAMUEL B. MORRIS, Chairman

JEWELL M. GARRELTS, Vice-Chairman

GLENN W. HOLCOMB

OLIVER W. HARTWELL

ERNEST W. CARLTON

DON M. CORBETT